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MODELING OF TRANSFER OF PARTICLES OF DIFFERENT SIZE BY A TWO-PHASE FILTRATION FLOW

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A mathematical model of transfer of solid particles by a two-phase filtration flow is suggested. The sizes of the pore channels and particles are described by the corresponding size distribution functions. The numerical solution is compared to the solutions in which the particles are represented by the same sizes.

Introduction. Transfer of particles through a porous medium has been the subject of repeated investigations. Consideration was given to the processes of transfer by single- and two-phase filtration flows [1-4]. The emphasis was on the phenomenon of particle sedimentation on the walls of pore capillaries and the related change in the filtration-capacity characteristics of a stratum. An analysis of the factors affecting particle absorption by the porous medium shows that the main contribution to this phenomenon is made by the sieve effect [5].

Our previous works describe mathematical models of transfer of particles of the same size by single – [6] and two-phase [7] filtration flows. Below we suggest a mathematical model of retention of particles of different size from a two-phase filtration flow by a porous body. By phases, oil and water are meant. It is assumed that both the particles and pore channels are described by the corresponding size distribution functions. A numerical analysis of the influence of the particle size distribution on the increase in filtration resistance is carried out. It is shown that allowance for the differences in particle sizes leads to significant differences in solutions of the problem.

Mathematical Model. It is assumed that the particle size distribution function $\Psi(l)$ is known and that the solid particles are transferred only by water. The porous medium is represented as two interpenetrating continua [8, 9], one of which is related to mobile liquids and particles and the other to those in the motionless state.

Let $m_1 = m_1(x, y, z, t)$ be a part of the void space occupied by mobile oil and water and $m_2 = m_2(x, y, z, t)$ be a part of the void space with motionless phases:

$$m_1 + m_2 = m \,. \tag{1}$$

Hereafter, the index 1 will denote quantities characterizing the first continuum, while the index 2 will pertain to the second continuum.

Let us write the mass-conservation equation of the phases and components for the first continuum in the form

$$\frac{\partial}{\partial t}(m_1 S_{i1}) + \text{div}\, \mathbf{V}_i = -q_i\,, \quad i = \text{o, w}\,; \tag{2}$$

$$\frac{\partial}{\partial t}(C_1 m_1 S_{\text{w}1}) + \text{div}\,(C_1 \mathbf{V}_{\text{w}} - D_u\, \text{grad}\, C_1) = -q_{\text{c}}\,. \tag{3}$$

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The equations of motion the phases can be written in the form of the generalized Darcy law:

$$\mathbf{V}_i = -\frac{K_i}{\mu_i}\,\mathrm{grad}\,(P)\,, \quad i = \mathrm{o, w}\,. \tag{4}$$

The conservation equations for the second continuum are

$$\frac{\partial}{\partial t}(m_2 S_{i2}) = q_i\,, \quad i = \mathrm{o, w}\,; \tag{5}$$

$$\frac{\partial}{\partial t}(C_2 m_2 S_{\mathrm{w}2}) = q_{\mathrm{c}}^{\eta}\,. \tag{6}$$

The saturations and the particle concentration in the first continuum are related to those in the second continuum by the apparent relations

$$m S_{\mathrm{o}} = m_1 S_{\mathrm{o}1} + m_2 S_{\mathrm{o}2}\,, \tag{7}$$

$$m S_{\mathrm{w}} = m_1 S_{\mathrm{w}1} + m_2 S_{\mathrm{w}2}\,, \tag{8}$$

$$C m S_{\mathrm{w}} = C_1 m_1 S_{\mathrm{w}1} + C_2 m_2 S_{\mathrm{w}2}\,. \tag{9}$$

To describe mass transfer between the two continua and changes in the filtration-capacity characteristics of the stratum due to colmatage, we use the equation for the pore size distribution function [6, 7]:

$$\frac{\partial \varphi}{\partial t} + U_r \frac{\partial \varphi}{\partial r} + U_\eta = 0\,, \tag{10}$$

where U_r is determined by the dependence [4, 7]

$$U_r = -C_1 S_{\mathrm{w}1}\left(\frac{2u_{\mathrm{a}} D^2}{rL}\right)^{1/3}\,. \tag{11}$$

The quantity U_η depends on the sizes of both the particles and the pore channels and is determined in the context of a model representation of the porous medium in the form of a bunch of capillaries of different radius in the same way as has been done in [6, 7]. With allowance for the particle size distribution function, we can write

$$U_\eta = -\frac{\beta C_1\,|V_{\mathrm{w}}|\,\eta r^2}{m_1}\,\frac{\int\limits_{2rh}^{\infty} \Psi(l)\,dl}{\frac{2}{3}\int\limits_{0}^{\infty} \Psi(l)\,l^3\,dl}\,. \tag{12}$$

The change in the absolute permeability caused by the change in the void space structure due to colmatage and pore clogging can be evaluated by representing the permeability for the current instant of time $k_1(x, y, z, t)$ in the form $k_1 = \bar{k}^0$, where $\bar{k}(x, y, z, t)$ will be determined by using the model of parallel capillaries and the Poiseuille equation:

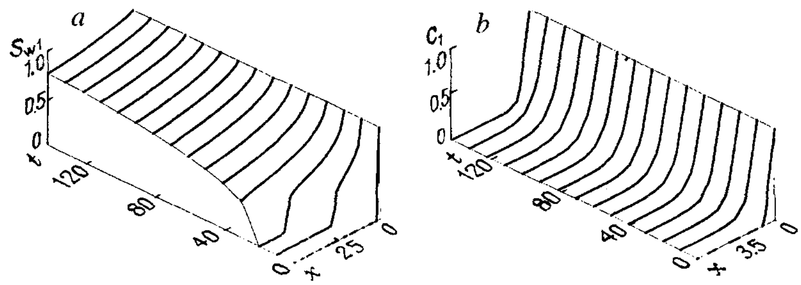

Fig. 1. Dynamics of the water saturation of the first continuum (a) and of the particle concentration in it (b).

$$\bar{k} = \int_0^\infty r^4 \eta(r)\, dr \Big/ \int_0^\infty r^4 \varphi^0(r)\, dr . \tag{13}$$

The intensity of the transition of the water from the mobile to the motionless (caused by clogging of the pore channels) state can be calculated from the share of the void space passed into the second continuum:

$$q_w = S_{w1} m_1 \int_0^{R/h} U_\eta r^2 dr \Big/ \int_0^\infty \eta(r)\, r^2\, dr . \tag{14}$$

For the oil, the intensity of the transition from the mobile to the motionless state will be assumed in the form

$$q_o = (1 - S_{w1})\, m_1 \int_0^{R/h} U_\eta r^2 dr \Big/ \int_0^\infty \eta(r)\, r^2\, dr . \tag{15}$$

The intensity of the transition of the particles to the motionless state due to clogging q_c^η is

$$q_c^\eta = C_1 q_w , \tag{16}$$

and the total intensity is

$$q_c = q_c^\eta + q_c^r , \tag{17}$$

where

$$q_c^r = -2m_1 \int_0^\infty \eta r U_r\, dr \Big/ \int_0^\infty \eta\, r^2\, dr . \tag{18}$$

Note that in the particular case where the particles have the same size, formula (9) coincides with the expression for the rate U_η obtained in [7].

Calculation Results. An algorithm of numerical solution of one-dimensional problems is developed on the basis of the finite-element method of reference volumes and finite-difference approximation with respect to time. Use is made of linear finite elements, the "lumping-approach," and upstream approximation of the convective terms entering in the filtration equation [10]. The discrete equations for the nodal values of the sought variables were solved by the direct method.

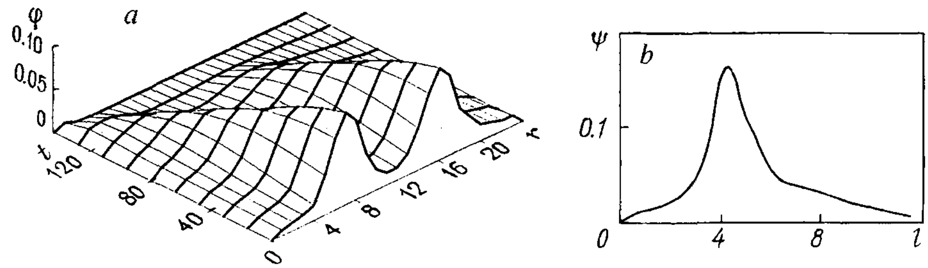

Fig. 2. Dynamics of the pore size distribution function at the stratum inlet (a) and the particle size distribution function (b).

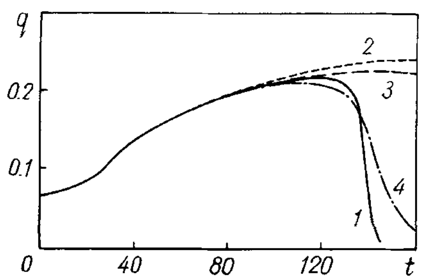

Fig. 3. Fluid flow rate through the stratum: 1) particles are described by the size distribution function; 2) particles with a size of 6 μm; 3) 8; 4) 12.

The influence of the disperse particles in water entering the stratum on the filtration characteristics was studied using the stratum of unit cross-sectional area as an example. The extension of the stratum was 50 m, the absolute permeability of the stratum (not clogged with particles) was 0.2 μm, and the porosity was 0.2. The viscosity of the water and the oil was 1 and 5, respectively. Functions of the relative phase permeabilities were taken in the form $f_o = (1 - S_{w1})^2$ and $f_w = (S_{w1})^2$. The stratum saturations with bound water and residual oil were assumed to be equal to 0.3. Water with solid particles, the volume concentration of which was 0.001 m^3/m^3, entered the stratum via the inlet section. The pressure drop in the stratum was 0.9 MPa. The maximum radius of the capillaries satisfying the clogging condition was 24 μm. It was assumed that the ratio of the radii of the throats to the radii of the pore channels was 0.25. Calculations were carried out for the pore size distribution function having two extrema at the 8- and 16-μm points. Figure 1 shows the dynamics of the pore size distribution function (a) calculated in the inlet section of the stratum with the use of the particle size distribution function given in the same figure. Figure 2 displays the dynamics of the water saturation (a) and concentration of solid particles (b) in the first continuum. It is seen that practically all the particles are retained by the porous medium over the first several meters. At a distance of 6 m, the particle concentration decreases by a factor of 10^4. Figure 3 illustrates the dynamics of the fluid flow rate via the stratum. Here, for comparison, calculation results are povided for the cases where the particles are of the same size. The graphs presented show that modeling of particles by one characteristic dimension must not rest on the size fitting a maximum of the particle size distribution function. The most similar results are obtained when the particles are represented by their maximum size.

NOTATION

m, porosity; m_1, dynamic porosity; m_2, part of the void space with the motionless liquid; **V**, total filtration rate; V_w, water-filtration rate; $\mathbf{V}_o$, oil-filtration rate; P, pressure; S_o, stratum saturation with oil; S_w, water saturation of the stratum; S_{oi}, saturation of the ith continuum with oil; $\underline{S}_{wi}$, water saturation of the ith continuum; k^0, absolute permeability; k_1, permeability of the first continuum; $\bar{k}$, coefficient characterizing the relative change in the permeability of the first continuum; K_i, phase permeability for the ith liquid; μ_w, dynamic viscosity of the water; μ_o, dynamic viscosity of the oil; C, concentration of the solid particles in the stratum; C_1, C_2, particle concentration in the first and second continua; r, radius of the pore channel; h, constant equal to the ratio of the radius of the throat to the radius of the pore channel; t, time; η, share of the capillaries of radius r; φ, pore size distribution function; φ^0, pore size distribution function at the initial instant of time; Ψ, particle size distribution function; U_r, rate of change in the pore-channel radii; U_η, rate of change in the number of pore channels of radius r; l, characteristic dimension of the particles; L, characteristic length of the pore channels; $R = l/h$; u_a, average fluid velocity in the channel; D, diffusion coefficient; D_u, convective diffusion coefficient; β, proportionality coefficient ($0 < \beta \le 1$); q_i, intensity of the transition of the ith liquid from the mobile to the motionless state; q_c^r, intensity of the transition of the particles from the mobile to the motionless state due to colmatage.

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